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Some Studies on Heuristic Implications of Robust Control Theory

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Abstract—*The main aim of this paper is to investigate robustness of DC motor under parametric uncertainty. In this paper the difference between robust aspect of control and compensation offered by phase lag controller on field controlled DC motor has been analyzed. The phase lag controller when applied to the system enhances the steady state response of the system but does not decimates the changes due to parameter variation, hence a robust controller along with a forward controller is applied to the system, which reduces the effect of parameter variation on the system response.*

Keywords*: Phase lag controller; Robust controller; DC motor; forward controller.*

1. INTRODUCTION

Robustness in the performance of a system is a difficult attribute to achieve. For instance, a phase lag compensator can improve the steady state response of a system, a phase lead compensator can improve the rise time and settling time of the system, but they cannot decimate the performance variation of the system due to changes in the parameter disturbance.

Robustness of a system is depicted from a fact that even if there is a peculiar change in one or more parameters of a system due to presence of uncertainties there is a negligible change in performance of the system. Uncertainties can be broadly classified into structured and unstructured. Structured uncertainties are variations which occur in poles, zeros and amplifier's gain of a particular system, while unstructured uncertainties occurs due to high frequency mode rejection in modeling of plants.

In this paper a controller is designed which would make the system (DC motor) inculcate property of a phase lag compensator and robustness. A comparison between response of power system stabilizer when controlled by PI, PD, PID and lead-lag compensator is performed in [1]. [5] Shows that performance variation due to uncertainties and disturbance can be decimated by the same controller. [3], [4], [5] contain basics of control engineering and robust control theory.[2] shows effect of right hand open loop poles and zeros on closed loop system's sensitivity.[5] and [6] depict the basics of robust control theory. In [12], [13], [15], and [16] robust aspect of control is utilized to achieve stability with respect to parameter variation for various systems. [10] And [11] dictate a robust and simple method for tuning a PID controller for integrator/dead time processes. [9] shows how kharitonov theorem can be applied to convert various polynomials generated by parameter variation into four polynomial which are enough to determine the stability of system. In [14] Arguon theorem is applied for checking stability of lower order polynomials with perturbation. [7] Contains the difference in attributes of internal model control and sliding mode when applied for vehicle yaw control. In [8] an internal model based neural control is applied on various discrete processes.

The transfer function relating the output and input of the system shown by the fig.(1) is-

$$M(s) = \frac{Y(s)}{R(s)} = \frac{KG_pG_rG_{cf}}{1 + KG_pG_r} \qquad (1)$$

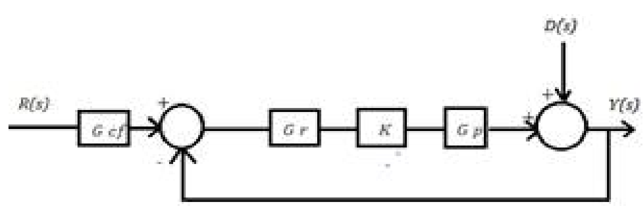

Fig.(1)- A closd loop control scheme

The relation between D(s) and Y(s) is given by-

$$N(s) = \frac{Y(s)}{R(s)} = \frac{1}{1 + KG_pG_r} \qquad (2)$$

Sensitivity of M(s) with respect to K is-

$$S = \frac{dM(s)/M(s)}{dK/K} = \frac{1}{1 + KG_pG_r} \qquad (3)$$

As the values of equation (2) and (3) are similar, thus a controller can be designed which could eliminate performance variation due to disturbance and changes in parameter [5]. In this paper we have described the difference between responses of a field controlled DC motor when compensated by phase lag compensator and when controlled by robust controller. This paper also shows the effect of a control scheme which contains a robust controller and a forward controller, on the system.

2. MODELING OF FIELD CONTROLLED DC MOTOR

Field controlled DC motor can be represented by the Fig.(2)-

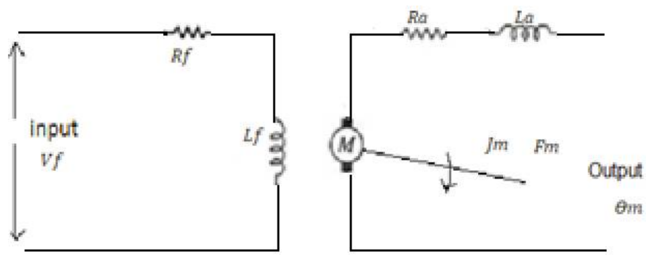

Fig.(2)-circuit model of a field controlled DC motor

Where 'V_f' is the voltage which is applied to field winding which controls the position of the shaft 'θ_m'.Applying kirchoff's voltage law in the circuit in fig.(2) we have-

$$V_f = i_f R_f + L_f \frac{di_f}{dt} \quad (4)$$

Taking laplace transform of equation (4)-

$$V_f(s) = i_f(s) R_f + L_f s i_f(s) \quad (5)$$

The motor torque developed is directly proportional to the input field voltage-

$$T_m \propto i_f \quad (6)$$

Or,

$$T_m = k_f i_f \quad (7)$$

The motor torqueTmcan be related with the angular position of the motor shaft $\theta_m$as-

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + F_m \frac{d\theta_m}{dt} \quad (8)$$

Taking the laplace transform of the above equation we get-

$$T_m(s) = J_m \theta_m(s) s^2 + F_m \theta_m(s) s \quad (9)$$

Combining the equations (5), (7), (9) we get the ratio of $\theta_m(s)$And $V_f(s)$ which gives the transfer function of the field controlled DC motor as-

$$A(s) = \frac{\theta_m(s)}{V_f(s)} = \frac{K_f}{s(J_m s + F_m)(R_f + L_f s)} \quad (10)$$

Values of constants are taken as-

K_f=Motor torque constant=0.025Nm/A.

J_m=moment of enertia of shaft=.0022$Nm/rad/sec^2$.

F_m=Coefficient of viscous friction

=0.003025 $Nm/rad/sec$

R_f =feild Resistance=1Ω

L_f =field inductance=0.01H.

Equation (10) can be written as-

$$A(s) = \frac{\theta_m(s)}{V_f(s)} = \frac{1136.36}{s(s^2 + 101.375s + 137.5)} \quad (11)$$

The closed loop shceme DC motor can be given as-

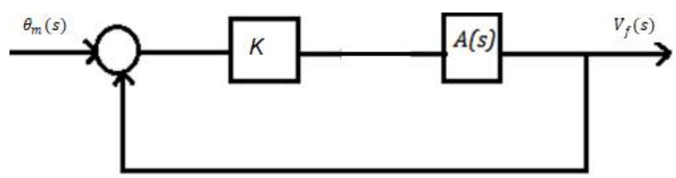

Fig.(3)-Closed loop scheme with DC motor model.

The closed loop transfer function with nominal value of K(=1) can be given as-

$$B(s) = \frac{\theta_m(s)}{V_f(s)} = \frac{1136.36}{s^3 + 101.375s^2 + 137.5s + 1136.36} \quad (12)$$

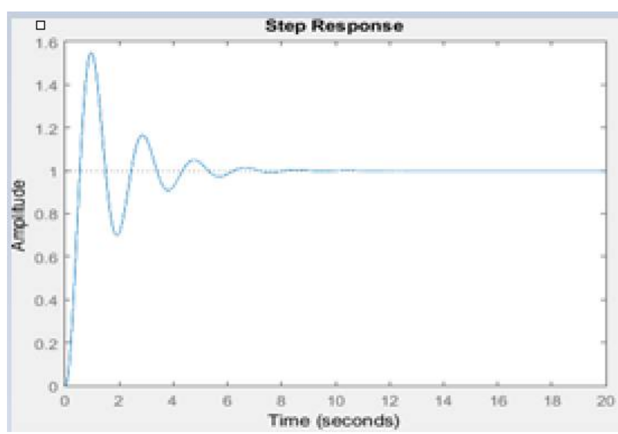

Fig. (4)-Step response of B(s)

In order improve the steady state respones of the system a phase lag controller of the form-

$$G_c(s) = \frac{1}{a}\frac{(1 + aTs)}{(1 + Ts)} \qquad (13)$$

with value of $a$and T as-0.1,30,along with$A(s)$ can be employed in a closed loop scheme to improve the steady state response of the dc motor.The control scheme is-

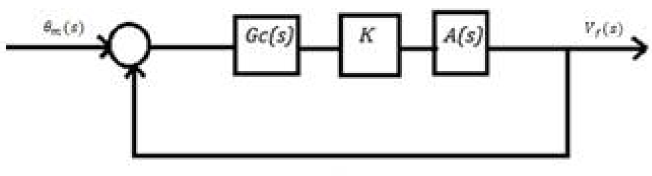

Fig. (5)-Block diagram of system with lag compensator.

The closed loop transfer function of the above system is given as-

$$P(s) = \frac{3409.08Ks + 1136.36K}{30s^4 + 3042.25s^3 + 4226.375s^2 + (3409.08K + 137.5)s + 1136.36K}$$

The step response of the above system with different values of$K$(0.5, 1, 1.5, 2) is-

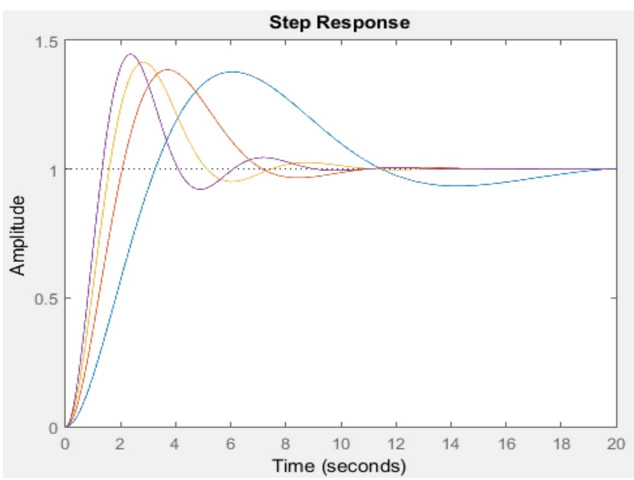

Fig. (6)-step responses for different values of K.

The attributes of unit step response system P(s) is given by the Table-1

Table-1

K	Rise Time	Settling Time	Overshoot	Roots of the characterstic equation.
0.5	3.25	17.97	37.77	-0.97,-100 -0.21+i0.38 -0.21-i0.38
1	2.05	9.64	38.61	-100.01,-0.5 -0.43+i0.72 -0.43-i0.72
1.5	1.58	9.51	41.56	-100.01,-0.4 -0.48+i1.04 -0.48-i1.04
2	1.32	8.28	44.57	-100.02,-.39 -0.49+i1.29 -0.49-i1.29

3. ROBUST CONTROLLER DESIGN

It is noticed from fig(6) that steady state response of the DC motor represented by $A(s)$is improved by applying $G_c(s)$, but variation in the response of the system due variation in K is not removed.Table-1, shows a clear change in values of roots of the system with changing value of amplifier's gain(K).To remove the performance variation a robust controller $G_r(s)$ is encorporated in fig(5) in place of $G_c(s)$.The control scheme in fig(5) changes to-

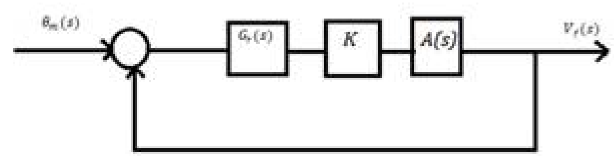

Fig. (7)-Closed loop scheme with robust controller.

Robust controller $G_r(s)$ is designed by placing the zeros of the controller near the desired closed loop poles as depicted in Table-1.The desired characteristic root for the nominal value of $K = 1$ is selected from the Table-1for the designing of the robust controller $G_r(s)$.

The roots selected are-(0.4+i.07);(0.4-i0.7).Thus robust controller is designed as-

$$G_r(s) = \frac{(s + 0.4 + i0.7)(s + 0.4 - i0.7)}{0.65} = \frac{s^2 + 0.8s + 0.65}{0.65} \qquad (14)$$

The closed loop transfer function of the system shown by fig(6) is given by-

$$Q(s) = \frac{1748.24Ks^2 + 1398.6Ks + 1136.36K}{s^3 + (101.375 + 1748.24K)s^2 + (137.5 + 1398.6K)s + 1136.36k} \qquad (15)$$

After application of robust controller the sensitivity of the DC motor with respectto variation in K is greatly improved.This is shown by Table-2.

Table 2

K	Roots of the characteristic equation of Q(s)
0.5	-974.637,-0.429+i0.632, -0.429-i0.632
1	-1848.78,-0.415i+0.665 -0.415-i0.65
1.5	-2722.91,-0.410+i0.676 -0.410-i0.676
2	-3597.03,-0.407+i0.682 -0.407-i0.682

As the roots of characterstic equation of $Q(s)$become similar for different values of K thus the robust controller deframes the effect of variation of K on the system.

4. FINAL CONTROL SCHEME DESIGN

As the zeros of the forward path transfer function of the control scheme shown in fig(7) are same as that of the closed loop transfer function, thus a controller should be designed which would remove the eventual canceling of the close loop zeros and poles.

The controller, also known as forward controller $[G_{cf}(s)]$ is designed as-

$$G_{cf}(s) = \frac{1}{G_r(s)} \quad (16)$$

Taking the value of $G_r(s)$ from the equation (14),the value of $G_{cf}(s)$ is given as-

$$G_{cf}(s) = \frac{0.65}{s^2 + 0.8s + 0.65} \quad (17)$$

The final control scheme inculcating the forward controller is given as-

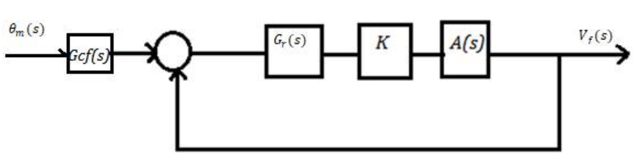

Fig.(8)-final control scheme with forward controller

The closed loop transfer function for the above system is given as-

$$T(s) = \frac{1136.36K}{s^3 + (101.375 + 1748.24k)s^2 + (137.5 + 1398.6K)s + 1136.6K} \quad (18)$$

The unit step response of the closed loop system shown in the Fig.(8) is given by Fig.(9)-

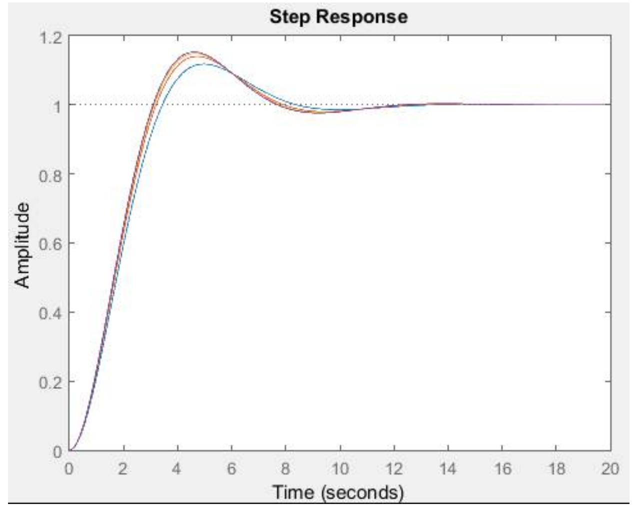

Fig. (9)-response of the DC motor with robust and forward controller.

Table-3 below shows the attributes of the closed loop system shown in the Fig.(8)-

Table-3

K	Rise Time	Settling Time	Overshoot	Root of the characteristic equation
0.5	3.43	7.67	11.83	-974.637 -0.43+i0.632 -0.43-i0.632
1	3.20	7.37	14.05	-1848.78 -0.42+i0.665 -0.42-i0.665
1.5	3.12	9.89	14.86	-2722.91 -0.41+i0.676 -0.41-i0.676
2	3.09	9.96	15.28	-3597.03 -0.41+i0.682 -0.41-i0.682

Table-3 and Fig.(9) show the effect of robust controller $G_r(s)$ on the system(DC motor).If Fig.(6) and Fig.(9) are compared, it can easily be noticed that the variation in performance of the system due to variation in the value of amplifier's gain(K),has been removed to a greater extent. The same can be experienced if we compare Table-1 and Table-3.

If we compare the response in Fig.(4) and Fig.(9), then it can be noticed that the steady state response of the system is also improved. This is due to fact that the robust controller was designed by considering the close loop poles of a system made by phase lag controller and the model transfer function of the DC motor[$A(s)$].

5. CONCLUSION

The system (DC motor) is modelled and its unit step response is generated. The system is then subjected to parameter variation and the influence of phase lag controller on the system is noticed. The phase lag controller enhances the steady state response of the system but is unsuccessful in reducing the sensitivity of the system with parameter variation. Hence a control scheme with robust controller and forward controller is designed which reduces the sensitivity of the system with parameter variation.

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